DEFINITION

Operations research (OR)

Operations research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.

The process of operations research can be broadly broken down into the following steps:

1. Identifying a problem that needs to be solved.
2. Constructing a model around the problem that resembles the real world and variables.
3. Using the model to derive solutions to the problem.
4. Testing each solution on the model and analyzing its success.
5. Implementing the solution to the actual problem.

Disciplines that are similar to, or overlap with, operations research include statistical analysis, management science, game theory, optimization theory, artificial intelligence and network analysis. All of these techniques have the goal of solving complex problems and improving quantitative decisions.

The concept of operations research arose during World War II by military planners. After the war, the techniques used in their operations research were applied to addressing problems in business, the government and society.

Characteristics of operations research

There are three primary characteristics of all operations research efforts:

1. Optimization- The purpose of operations research is to achieve the best performance under the given circumstances. Optimization also involves comparing and narrowing down potential options.
2. Simulation- This involves building models or replications in order to try out and test solutions before applying them.
3. Probability and statistics- This includes using mathematical algorithms and data to uncover helpful insights and risks, make reliable predictions and test possible solutions.

Importance of operations research

The field of operations research provides a more powerful approach to decision making than ordinary software and data analytics tools. Employing operations research professionals can help companies achieve more complete datasets, consider all available options, predict all possible outcomes and estimate risk. Additionally, operations research can be tailored to specific business processes or use cases to determine which techniques are most appropriate to solve the problem.

Uses of operations research

Operations research can be applied to a variety of use cases, including:

- Scheduling and time management.
- Urban and agricultural planning.
- Enterprise resource planning (ERP) and supply chain management (SCM).
- Inventory management.
- Network optimization and engineering.
- Packet routing optimization.
- Risk management.

Linear Programming Problem

The LPP technique was first introduced in 1930 by Russian mathematician Leonid Kantorovich in the field of manufacturing schedules and by American economist Wassily Leontief in the field of economics. After a decade during World War II, these techniques were heavily adopted to solve problems related to transportation, scheduling, allocation of resources, etc. In 1950, the first simplex method algorithm for LPP was created by American mathematician George Dantzig.

Elements of a basic LPP

Decision Variables: These are the unknown quantities that are expected to be estimated as an output of the LPP solution.

Objective Function: All linear programming problems aim to either maximize or minimize some numerical value representing profit, cost, production quantity, etc. It evaluates the amount by which each decision variable would contribute to the net present value of a project or an activity.

Objective Function coefficient: The amount by which the objective function value would change when one unit of a decision variable is altered, is given by the corresponding objective function coefficient.

Constraints: The restrictions or limitations on the total amount of a particular resource required to carry out the activities that would decide the level of achievement in the decision variables. In the standard form of a linear programming problem, all constraints are in the form of equations.

Non-negative constraints: Each decision variable in any Linear Programming model must be positive irrespective of whether the objective function is to maximize or minimize the net present value of an activity. This is a critical restriction.

The other two elements are Resource availability and Technological coefficients which can be better discussed using an example below. A feasible solution to the linear programming problem should satisfy the constraints and non-negativity restrictions. A feasible solution to an LPP with a maximization problem becomes an optimal solution when the objective function value is the largest (maximum). Similarly, a feasible solution to an LPP with a minimization problem becomes an optimal solution when the objective function value is the least (minimum).

- Maximize 15*x1 + 10*x2; subject to
- 0.25*x1 + 1*x2 ≤ 65
- 1.25*x1 + 0.5*x2 ≤ 90, where x1 & x2 ≥ 0

Decision variables

x1 and x2

Objective function

Maximize 15*x1 + 10*x2

Objective function coefficients

15: x1 and 10: x2

Constraint-1

0.25*x1 + 1*x2 ≤ 65

Constraint-2

1.25*x1 + 0.5*x2 ≤ 90

Non-negative constraints

x1 ≥ 0 & x2 ≥ 0

Technological coefficients

0.25	1
1.25	0.50

Resource availability

65
90

Decision variables	Number of constraints in primal define the number of decision variables of the dual. Primal has two constraints, so dual can have two decision variables (y1 & y2)
Objective function	If the primal has to maximize then dual has to be for minimize. Similarly if the primal has to minimize then dual has to be for maximize
Objective function coefficients	The resource availability of primal becomes the objective function coefficient of the dual. 65: y1 and 90: y2
Resource availability	The objective function coefficients of the primal becomes the resource availability of the dual. Therefore dual resource availability becomes [15, 10]
Technological coefficients	The technological coefficients of the primal gets transposed and becomes the technological coefficients of the dual.
Non-negative constraints	If the decision variables of primal are non-negative, then decision variables of dual also become non-negative
Constraints	Number of decision variables in primal define the number of constraints in the dual. Primal has two decision variables, so dual can have two constraints. The sign of the constraints would reverse

Operations Research Phases

Following are the six phases and processes of operational research:

Formulate the problem: This is the most important process, it is generally lengthy and time consuming. The activities that constitute this step are visits, observations, research, etc. With the help of such activities, the O.R. scientist gets sufficient information and support to proceed and is better prepared to formulate the problem.

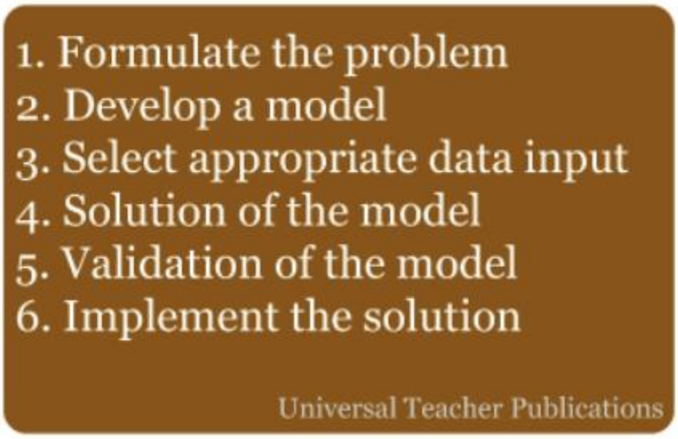
This process starts with understanding of the organizational climate, its objectives and expectations. Further, the alternative courses of action are discovered in this step.

Develop a model: Once a problem is formulated, the next step is to express the problem into a mathematical model that represents systems, processes or environment in the form of equations, relationships or formulas. We have to identify both the static and dynamic structural elements, and device mathematical formulas to represent the interrelationships among elements. The proposed model may be field tested and modified in order to work under stated environmental constraints. A model may also be modified if the management is not satisfied with the answer that it gives.

Select appropriate data input: Garbage in and garbage out is a famous saying. No model will work appropriately if data input is not appropriate. The purpose of this step is to have sufficient input to operate and test the model.

Solution of the model: After selecting the appropriate data input, the next step is to find a solution. If the model is not behaving properly, then updating and modification is considered at this stage.

Validation of the model: A model is said to be valid if it can provide a reliable prediction of the system's performance. A model must be applicable for a longer time and can be updated from time to time taking into consideration the past, present and future aspects of the problem.



The Graphical Method

We will first discuss the steps of the algorithm:

Step 1: Formulate the LP (Linear programming) problem

We have already understood the mathematical formulation of an LP problem in a previous section. Note that this is the most crucial step as all the subsequent steps depend on our analysis here.

Browse more Topics under Linear Programming

- Different Types of Linear Programming Problems
- Linear Programming Problem and Its Mathematical Formulation

Step 2: Construct a graph and plot the constraint lines

The graph must be constructed in ‘n’ dimensions, where ‘n’ is the number of decision variables. This should give you an idea about the complexity of this step if the number of decision variables increases.

One must know that one cannot imagine more than 3-dimensions anyway! The constraint lines can be constructed by joining the horizontal and vertical intercepts found from each constraint equation.

Step 3: Determine the valid side of each constraint line

This is used to determine the domain of the available space, which can result in a feasible solution. How to check? A simple method is to put the coordinates of the origin (0,0) in the problem and determine whether the objective function takes on a physical solution or not. If yes, then the side of the constraint lines on which the origin lies is the valid side. Otherwise it lies on the opposite one.

Step 4: Identify the feasible solution region

The feasible solution region on the graph is the one which is satisfied by all the constraints. It could be viewed as the intersection of the valid regions of each constraint line as well. Choosing any point in this area would result in a valid solution for our objective function.

Step 5: Plot the objective function on the graph

It will clearly be a straight line since we are dealing with linear equations here. One must be sure to draw it differently from the constraint lines to avoid confusion. Choose the constant value in the equation of the objective function randomly, just to make it clearly distinguishable.

Step 6: Find the optimum point

Optimum Points

An optimum point always lies on one of the corners of the feasible region. How to find it? Place a ruler on the graph sheet, parallel to the objective function. Be sure to keep the orientation of this ruler fixed in space. We only need the direction of the straight line of the objective function. Now begin from the far corner of the graph and tend to slide it towards the origin.

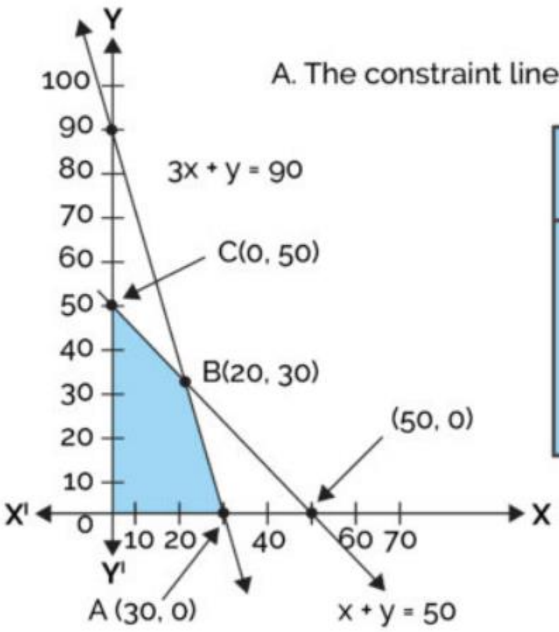
- If the goal is to minimize the objective function, find the point of contact of the ruler with the feasible region, which is the closest to the origin. This is the optimum point for minimizing the function.
- If the goal is to maximize the objective function, find the point of contact of the ruler with the feasible region, which is the farthest from the origin. This is the optimum point for maximizing the function.

Solved Example

Q. Maximize and minimize $z = 4x + y$ subject to:- $x + y \le 50$

$3x + y \le 90$

$x \ge 0, y \ge 0$



A. The constraint lines are $x + y = 50$, $3x + y = 90$, $x = 0$, $y = 0$

Corner Point	Corresponding value of Z
(0, 0)	0
(30, 0)	120
(20, 30)	110
(0, 50)	50

Hence, maximum value of Z is 120 at the point (30, 0) and the minimum value of z is 0 at the point (0, 0).